You Won the Battle. What about the War? A Diffusion Model on Proprietary–Open Source Software Competition

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Abstract

Although a relevant body of the economics literature is increasingly concentrating on the issue of Open Source Software (OSS), a formal treatment concerning the process of competition between proprietary software and its open source counterpart is still missing. Relying on a well developed stream of the literature pertaining to the diffusion of innovation, we try to overcome this gap. In particular, we propose a model where the two technologies depend on a set of different factors, each one specific to its own mode of production: profits and developers' motivations respectively. At the same time, both proprietary and open source software diffusion are made to depend on the presence of network effects and the level of interoperability. Moreover, the possibility for a joint use of both softwares is taken into consideration and properly modeled. In addition, compared to standard diffusion models, we introduce a technical innovation endogenizing the parameter influencing the speed of diffusion across the population of adopters. Main results are presented by means of a comparative dynamics exercise.

Keywords: Increasing returns; Technology diffusion; Technological competition; Open-source software.
JEL Classification: L17; L86; O33

1 Introduction

The outstanding growth of adoption of Open Source Software (OSS) in recent years has attracted the attention of many scholars in different fields of study. Indeed, a large number of successful case studies have been presented so far in order to justify and ground empirically such a relevant gain in popularity and this led to a proliferation of studies in many directions of enquire. Among the others, topics such as the organisation and ethos of the community of developers, together with their motivation to provide code for free and the birth of hybrid business models have been extensively examined by different branches of social science literature. An issue that has not been sufficiently covered by the literature and that needs particular attention is the one pertaining to the OSS innovation model and the way in which it interacts with the Proprietary Software (PS) one. Some studies have been put forward by the literature on the topic[1], but they have been extensively lacking some sort of formal treatment. To our knowledge, only a few contributions have tried to overcome such a shortfall, that is Bonaccorsi and Rossi (2003); Dalle and Jullien (2003). Nevertheless, all of them have not relied on a branch of the literature of economics of innovation that, we think, provides useful insights on the modeling task to be carried out, i.e. innovation diffusion models.

This is why our work starts from a fruitful integration of the literature dealing with the process of competition between OSS and PS with the one pertaining to the models of innovation diffusion, in particular epidemic models. The starting point is to model the process of competition arising between OSS and PS by means of a system of non-linear differential equations. In particular, the model takes into account both demand and supply sides of the two different technologies, together with the extent of competition between the two. Relying on the literature of OSS, we make technologies depending on different sets of factors. In particular, the demand and the supply of OSS respond to reputational and communitarian factors (intrinsic and extrinsic motivations) while those of PS are essentially profit oriented. At the same time, both PS and OSS diffusion are influenced by the presence of network effects and the level of interoperability. Moreover, the possibility for a joint use of both softwares is taken into consideration and properly modeled. Finally, compared to standard diffusion models (Hofbauer and Sigmund, 1998), we introduce a technical innovation endogenizing the parameter influencing the speed of diffusion across the population of adopters.

The remainder of the paper is structured as follows. First, we discuss the theoretical background pertaining to the issue at stake. In particular, available results in the area of both competition between proprietary and

[1] For a comprehensive review of the literature dealing with all of the above mentioned topics see Rossi (2006).

open source software (section 2.1) and innovation diffusion models (section 2.2) are put forward. On the grounds of the previously mentioned results, we will discuss the general framework in which the two competing technologies must be conceived, that is the software industry. In particular, a set of three main features of software technology will be considered: network effects, interoperability and joint adoption. These characteristics are at the core of the formal model therein introduced (section 3). In particular, several variants of the model will be discussed: (i) a base version characterised by constant propagation coefficients of the two technologies (section 3.1); (ii) an extended version incorporating changing propagation coefficients both under perfect (section 3.2.1) and non-perfect interoperability (section 3.2.2). Finally, section 4 briefly concludes.

2 Theoretical Background

2.1 The Competition between Proprietary and Open Source Software

The issue of Open Source Software (hereafter OSS) has gained momentum in recent years thanks to the echo derived from a number of relevant "success stories". Firefox among internet browsers, Apache among web servers, Linux in the operating system market and Openoffice among office suites are all well known examples. This increasing notoriety should be explained by three main stylized facts. First of all, OSS has questioned the traditional process of software development as a proprietary one. Indeed, recent years have witnessed to continuous erosion of the user base of proprietary solutions by main OSS products. Second, a discrete number of large firms have decided to enter into the market for software in order to challenge Microsoft's monopolistic position. To do so, they have heavily relied on the competitive advantage constituted by OSS. Among the others, IBM, Novell and Red Hat are worth mentioning for their outstanding performance in this field. This fact has brought to the creation of a new type of business model in the software sector, i.e. the hybrid model of software production. This is characterised by the presence of private companies providing OSS solutions, which are mainly developed by OSS communities. Third, an increasing number of countries all over the world has started to discuss about the role that FLOSS should have in Public Administration. Given the different nature of the public administration compared to private companies, many governments have began to ask whether OSS should be a useful way to save on governmental budgets and, at the same time, spurring local development.

At the same time, social science literature started to devote attention to the topic as well. In particular, contributions in this field can be organised into five general groups: (i) the part relative to the nature of developers' motivations; (ii) the problem of the governance of OSS projects; (iii) OSS relationships with the surrounding environment; (iv) the complex topic of IPRs' influence on OSS and (v) government policies towards OSS. A large part of the literature has mainly concentrated on the first two areas of interest, disregarding almost entirely the other topics.

In this section we briefly discuss findings of the third point in the list[2]. This branch of the literature has mainly concentrated on the relationship between OSS and proprietary software devoting particular interest to the process of competition between OSS and proprietary software production mode. The contributions in this field can be grouped in two main typologies. First, several contributions adopt a spatial competition model,*à la* Hotelling, to study the competition between proprietary software and open source assuming heterogeneity among software users' needs. In particular, Kuan (2001) stresses the importance that consumers have in the production process of OSS (Lakhani and Von-Hippel, 2003). This spurs her to model the competing process between the two modes of production assuming that agents must decide between buying software and producing it. Johnson (2002) models the decision of individual user-programmers to contribute to software program that, if developed, will result in a public good. He shows that programmer participates only if the benefit-cost ratio is higher than a certain threshold which increases with the probability of free-riding. Schmidt and Schnitzer (2003) identify three main groups of adopters: (i) consumers already using OSS; (ii) consumers already using proprietary software; (iii) users that choose between the two. The authors show that increasing the number of OSS users by means of public provided subsidies leads to an increase for software price of locked-in proprietary software users. Bitzer (2004) shows that product heterogeneity is the main factor explaining the ability of incumbent firm (the one adopting a proprietary mode of production) to be profitable by setting a higher price strategy than OSS entering firm. Bessen (2004) constructs a model in which the choice of the form of provision of software is endogenous. Free riding is a less pressing concern, provided that the base product is created in the first place. This is mainly due to the fact that the low probability that a specific feature will be developed by another consumer reduces the incentives to wait. As a result, the OSS form of software provision is more efficient because it allows the complex and sophisticated needs of some consumers to be fulfilled in markets where incomplete contracts and asymmetric information prevent proprietary software from serving all applications. Thus, a coexistence of the two software technologies is the norm.

Second, some other contributions have highlighted the role played in the software market by increasing returns on the demand side and, thus, they have tried to model network effects (both direct and indirect) which

[2]For a survey dealing with the main achievements of all the above mentioned literature strands, please see Rossi (2006).

might induce path-dependence processes (Arthur, 1989) and are likely to produce lock-in effects (David, 1985). For example, Bonaccorsi and Rossi (2003) adopt a collective action model and run simulations on a specific explicit OSS adoption function. Results yields that under some plausible assumptions both software production modes are likely to coexist in the future. Dalle and Jullien (2003) take into account network effects as well, but distinguish between local and global ones. The former refers to the proportion of a user's neighbors who have already adopted OSS, the latter to the proportion of adopters in the whole population. Then, they run simulations on the OSS adoption function incorporating these factors, together with other more standard ones, and find that the pace of code improvement and proselytism are important factors in explaining OSS success and its coexistence with proprietary software technology.

Thus, the literature briefly surveyed shows that, although differing with respect to main theoretical hypothesis (perfect vs bounded rationality, static model vs dynamic ones, etc) and to the methodology adopted (maximisation vs simulation), one common conclusion is that proprietary software and OSS are likely to coexist in the long-run.

2.2 The Diffusion of Innovation

Starting from Mansfield (1961) seminal contribution, a growing amount of literature has explored the characteristics of the process of diffusion[3]. In particular,Metcalfe (1981) and Batten (1987) have correctly argued that to understand the process of innovation diffusion a mechanism explaining the dynamics of the supply side must be incorporated into the demand-led epidemic model. In this way, the dynamic path follows a logistic pattern that is determined by the joint dynamics of market demand and growth in production capacity. Contrary to the standard diffusion models, in which the supply of a new technique is perfectly elastic and allows supply to adjust smoothly to growth in demand, these models show how the innovator's reward decreases during the diffusion process. Models of this kind give an evolutionary perspective of Schumpeterian competition among agents which is essentially characterized by the struggle of the innovator to overcome the system's resistance to novelty in order to gain differential profitability from the innovation. At the core of the epidemic approach there is the idea that the characteristics of a technology are subject to a progressive path of discovery. The characteristics of the new technique are not well known, and, as the available information spreads, the level of uncertainty associated with the new technique decreases, thus increasing the number of adopters. Also in more orthodox models, with both one (Karshenas and Stoneman, 1993) and multiple (Stoneman and Kwon, 1994) competing technologies, epidemic

[3]For a comprehensive survey on models of technology diffusion reporting a set of different models' typologies see Karshenas and Stoneman (1995); Geroski (2000).

effects have been introduced to explain the process of endogenous learning as a process through which information about a technology propagates as that technology spreads in the system. A further step in the direction of broader models of diffusion has been that of including more than one technique in order to show how the process of diffusion might be the result of competition among techniques, rather than the smooth diffusion of one only (Amable, 1992; Leoncini, 2001).

All the above mentioned models, however, can not be fruitfully implemented into the analysis of a particular technology such as software. Important adaptations should be implemented if a correct description of the industry wants to be carried out. In particular, phenomena idiosyncratic to the software mode of production should be added, that is network effects and possibility of joint adoption. Moreover, the general structure of epidemic models lacks in flexibility, generally assuming a constant coefficient of technology propagation. Hence, the model we propose in the next section goes exactly in this direction, overcoming above mentioned deficiencies related to the particular technology analysed. In addition, we improve the standard model by letting the coefficient to be endogenously determined by supply factors specific to each technology.

3 Model General Structure

First of all, we assume that the maximum number of potential software users (D) is exogenously given. Each software technology diffuses following a logistic pattern. In line with the epidemic approach, every current user of each technology has a given probability to persuade each current non-user to adopt it. The effectiveness of the "word of mouth" ($\beta_i > 0$) can be different for the competing technologies and is a function of the features of the technology itself.

As for PS (x), we assume that it is a strictly decreasing and concave function of its price (p):

$$\beta'_x(p) < 0$$

$$\beta''_x(p) \geq 0$$

Moreover, given that software industry is characterized by economies of scale, both static and dynamic (Shy, 2001), we assume further:

$$p'(x) < 0$$

$$p''(x) > 0$$

Hence, the speed of diffusion of PS is a strictly increasing and concave function of its level of adoption:

$$\beta'_x(x) > 0 \tag{1}$$

$$\beta_x''(x) < 0 \tag{2}$$

As for OSS (y), given that its development does not entail any explicit monetary cost, but it is simply the result of the efforts made by the community of developers responding both to intrinsic and extrinsic motivations (Lerner and Tirole, 2002; Bitzer, Schrettl, and Schroeder, 2007), its final users do not face any direct explicit adoption cost, but only implicit costs. Relying on the fact that, the more the efforts of the community are, the more the OSS is "developed" and therefore the less the costs that final users will suffer for using it will be, we assume a negative relation between the total amount of such efforts and the level of these costs. The probability of OSS adoption (β_y) is thus modeled as a strictly increasing function of community's efforts (e):

$$\beta_y'(e) > 0$$

Arguing from the decreasing marginal returns of efforts, we set further:

$$\beta_y''(e) \leq 0$$

Given that the sets of OSS final users and developers are likely to overlap due to the importance in the open source method of production of user-driven innovation (see section 2.1 and Von-Hippel and Von-Krogh (2003)), we assume that the level of efforts is positively related to that of OSS adoption (y):

$$e'(y) > 0$$

Moreover, given that larger communities of developers are more likely to face coordination problems, like disagreement on the actual piece of code to be incorporated into the final release, disputes over credit attribution and a higher probability of "forking" (Lerner and Tirole, 2002), we assume further that the increase of efforts in development is less proportional than the increase of the level of adoption:

$$e''(y) < 0.$$

Hence, the speed of diffusion of OSS is a strictly increasing and concave function of its level of adoption:

$$\beta_y'(y) > 0 \tag{3}$$

$$\beta_y''(y) < 0 \tag{4}$$

Each current user of technology i has a constant probability in each period ($0 \leq \theta_i \leq 1$) to use jointly the competing technology. Let θ_x (θ_y) be the probability that a user of PS (OS) is willing to use jointly the competing software. In particular, θ_y is influenced by the extent of free-ware/shareware applications and/or marketing strategies proprietary company can put into action in order to improve its reputation. On the contrary, θ_x is influenced by the presence of open source live distributions which might convince an adopter of PS to use also OSS thanks to the promise that the latter will not hurt its already installed software.

Finally, the pattern of diffusion is also affected by negative network effects (Katz and Shapiro, 1994; Liebowitz and Margolis, 1994). Such effects are modeled by considering negative interaction effects linked to the relative level of adoption of the concurrent technologies. In particular, we assume that each current user of technology i, who do not jointly use the competing technology j, has a constant probability to persuade each user of technology j to dismiss such technology. This probability ($\eta_i \geq 0$) is greater the greater is the lack of interoperability of technology i with technology j. Factors affecting the extent of network effects in the diffusion process might be fruitfully reconnected to the presence of non perfect interoperability between the two technologies.

Given the previous assumptions, the dynamics of diffusion can be thus represented by the following autonomous non-linear system of differential equations:

$$\begin{aligned} \frac{dx}{dt} &= \beta_x(x)\ x\ (D - x - (1-\theta_y)y) - \eta_x(1-\theta_y)\ x\ y \\ \frac{dy}{dt} &= \beta_y(y)\ y\ (D - (1-\theta_x)x - y) - \eta_x(1-\theta_x)\ x\ y \end{aligned} \tag{5}$$

3.1 Diffusion patterns with a constant propagation coefficient

Let us first assume that the actual level of technology diffusion does not significantly affect the relevant features of the technology, that is, the price for PS and the level of development for OSS ($p'(x) = e'(y) = 0$); or, equivalently, that such features do not alter the probability of adoption ($\beta'_x(p) = \beta'_y(e) = 0$)). Thus, we have that β_i ($i = x, y$) is a constant and the system (5) can be written:

$$\begin{aligned} \frac{dx}{dt} &= \beta_x\ x\ \left(D - x - (1+\frac{\eta_x}{\beta_x})(1-\theta_y)\ y\right) \\ \frac{dy}{dt} &= \beta_y\ y\ \left(D - (1+\frac{\eta_y}{\beta_y})(1-\theta_x)\ x - y\right) \end{aligned} \tag{6}$$

Equations (6) are the well-known Lotka-Volterra equations for two competing species (see, for instance, Hofbauer and Sigmund, 1998, Ch.3). The isoclines are straight lines with negative slopes:

$$y = \frac{D}{(1+\frac{\eta_x}{\beta_x})(1-\theta_y)} - \frac{1}{(1+\frac{\eta_x}{\beta_x})(1-\theta_y)}x \tag{7}$$

$$y = D - (1+\frac{\eta_y}{\beta_y})(1-\theta_x)\ x \tag{8}$$

Given the constraints on the parameters, these lines intersect at most once provided that $\frac{\theta_j}{1-\theta_j} \neq \frac{\eta_i}{\beta_i}$ for at least one technology. Figure 1 depicts the possible cases.

The sufficient and necessary condition for the stable coexistence of the technologies in the market is therefore that the following inequality holds for both technologies[4]

$$\frac{\theta_j}{1-\theta_j} > \frac{\eta_i}{\beta_i} \tag{9}$$

that is, the odds of adoption for joint use of technology i by users of technology j should be greater than the probability of dismissal for lack of interoperability of technology i with j divided by the probability of adoption of i (Figure 1(a)).

If condition (9) does hold for only one technology, this technology displaces completely the other (Figures 1(c) and 1(d)).

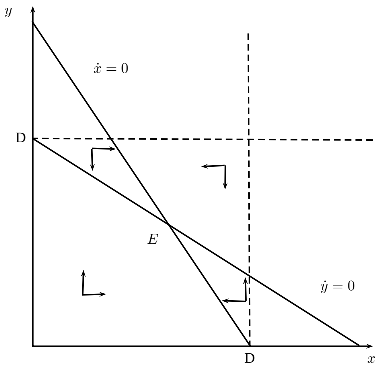
When instead condition (9) does not hold for any technology, we are in the so called *bistable case* (Figure 1(b)). There are two basins of attraction: the orbits in the first one converge to $(D, 0)$, whereas the others to $(0, D)$, while E is a saddle point. In such case the initial conditions actually matters.

Let us note that, in case of stable coexistence, an increase in the probability of instantaneous joint use of technology j by previous users of technology i decreases (increases) the *total* number of users of technology i (j), both exclusive and co-users; whereas an increase of the interoperability of technology i actually makes these users increase (decrease). In formal terms, if condition (9) is satisfied and the probability of joint use is not degenerate ($\theta_i < 1$) for both technologies, we have:

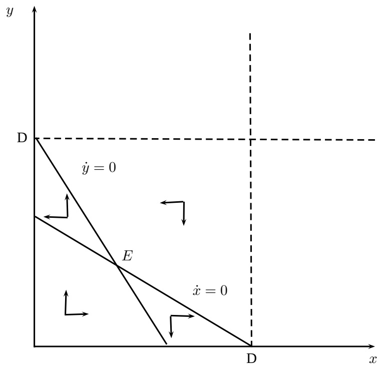
$$\begin{array}{llll}
\frac{\partial x^*}{\partial \theta_x} < 0; & \frac{\partial y^*}{\partial \theta_x} > 0; & \frac{\partial y^*}{\partial \theta_y} < 0; & \frac{\partial x^*}{\partial \theta_x} > 0; \\
\frac{\partial x^*}{\partial \beta_x} > 0; & \frac{\partial y^*}{\partial \beta_x} < 0; & \frac{\partial y^*}{\partial \beta_y} > 0; & \frac{\partial x^*}{\partial \beta_x} < 0; \\
\frac{\partial x^*}{\partial \eta_x} < 0; & \frac{\partial y^*}{\partial \eta_x} > 0; & \frac{\partial y^*}{\partial \eta_y} < 0; & \frac{\partial x^*}{\partial \eta_x} > 0.
\end{array}$$

So, for instance, if θ_x increases because of the reduction of the implicit cost of joint adoption for PS users of OSS technology, there is an absolute increase of OSS users (from $y*$ to $y'*$ in figure 2(a)) and a decrease of PS users (from $x*$ to $x'*$ in figure 2(a)). On the contrary, if the degree of interoperability of OSS technology decreases because of the closure of the

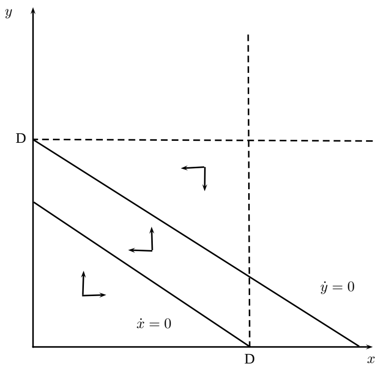
[4] Let us note that this equilibrium is globally stable (or uniformly asymptotically stable in the large). Thus, the initial conditions do not actually matter. For a proof of the global stability of the equilibrium in the case of stable coexistence for the Lotka-Volterra equation for two competing species by means of the Lyapunov function see, for instance, Gandolfo (1996).



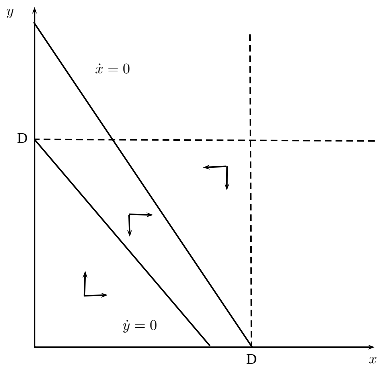
(a) High interoperability/probability of joint use for both



(b) Small interoperability/probability of joint use for both



(c) Small interoperability/probability of joint use for PS



(d) Small interoperability/probability of joint use for OSS

Figure 1: Dynamics with constant propagation coefficient

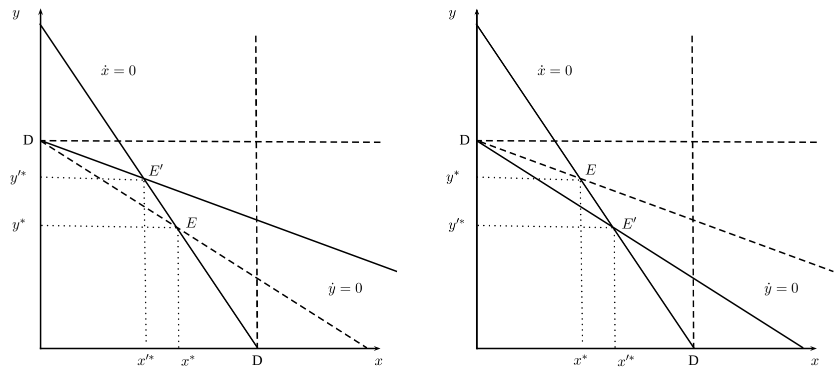
standards adopted by PS, this makes total OSS users decrease, whereas PS users instead increases (Figure 2(b)).

Let us finally note that the market shares in case of stable coexistence are not affected by the absolute size of the market.[5]

[5] The ratio between the total number of OS and PS users is indeed given by

$$\frac{y}{x} = \frac{\beta_x \left(\eta_y(1-\theta_x) - \beta_y \theta_x\right)}{\beta_y \left(\eta_x(1-\theta_y) - \beta_x \theta_y\right)}.$$

which does not depend on the total amount of demand (D).



(a) Reduction of the implicit cost of joint use of PS-OSS technologies for previous exclusive PS users

(b) Closure of PS standards

Figure 2: Comparative dynamics

3.2 Diffusion patterns with a changing propagation coefficient

3.2.1 Perfect interoperability

To begin with, let us study the case of perfect interoperability, that is where no network effects are present ($\eta_x = \eta_y = 0$). In such case Equations (5) become:

$$\begin{aligned} \frac{dx}{dt} &= \beta_x(x)\ x\ (D - x - (1-\theta_y)y) \\ \frac{dy}{dt} &= \beta_y(y)\ y\ (D - (1-\theta_x)x - y) \end{aligned} \tag{10}$$

Although β_i is now a function of the actual level of diffusion of the correspondent technology, the equilibrium values depend only on the probabilities of instantaneous joint use (θ_i) and are equal to:

$$\begin{aligned} x^* &= \frac{\theta_y D}{\theta_x + \theta_y - \theta_x\theta_y} \\ y^* &= \frac{\theta_x D}{\theta_x + \theta_y - \theta_x\theta_y} \end{aligned} \tag{11}$$

Provided that these probabilities are not degenerate, such equilibrium is asymptotically stable, no matter the actual form of the functions $\beta_i(i)$ (see Appendix B.1 for a proof of the asymptotical stability of the equilibrium). In such case, the situation is still the one depicted in Figure 1(a), although now there is no problem of interoperability. Thus, in the present case, the outcome is always the stable coexistence.

It is worth stressing that the market shares of the software technologies are not affected by those features that interact with the propagation coefficient – the price for PS and level of development for OS –, but only by the ones which instead affect the probability of instantaneous joint adoption. Thus, by assuming perfect interoperability and the possibility of joint use, the final outcome is necessarily neither the most efficient one nor the one in which the product with the best features (effective or potential) is actually chosen (Arthur, 1989).

3.2.2 Non perfect interoperability

In order to analyze the dynamics in the most complex case, we work out the isoclines:

$$y^*(x) = \frac{1}{1-\theta_y}\frac{\beta_x(x)}{\eta_x+\beta_x(x)}(D-x) \tag{12}$$

$$x^*(y) = \frac{1}{1-\theta_x}\frac{\beta_y(y)}{\eta_y+\beta_y(y)}(D-y) \tag{13}$$

Let us note first that the convex hull of $\{(0,0),(D,0),(D,D),(0,D)\}$ is the only relevant area, given that the threshold D is a physical constraint: the actual number of users of each technology cannot be greater than the maximum feasible number of users. Hence, we have to take into account only the interval $[0,D]$ for each variable.

To start with, we analyze the isocline of x in such interval. For Equation (12), we have:

$$\frac{dy^*}{dx} = \frac{1}{(1-\theta_y)\left(\eta_x+\beta_x(x)\right)^2}\left(\eta_x\beta_x'(x)(D-x)-\beta_x(x)(\eta_x+\beta_x(x))\right) \tag{14}$$

$$\frac{d^2y^*}{dx^2} = -\frac{2\eta_x(D-x)}{(1-\theta_y)(\eta_x+\beta_x(x))^2}\left(\frac{\beta_x'(x)^2}{\eta_x+\beta_x(x)}+\frac{\beta_x'(x)}{D-x}-\frac{\beta_x''(x)}{2}\right) \tag{15}$$

If conditions (1) and (2) hold and θ_y is not degenerate, we have $d^2y^*/dx^2<0$ for $x\in[0,D]$. Hence, Equation (12) is strictly concave in such interval. Moreover, given that:

$$\left.\frac{dy^*}{dx}\right|_{x=D} = -\frac{1}{1-\theta_y}\frac{\beta_x(D)}{\eta_x+\beta_x(D)}\ (<0)$$

and

$$\lim_{x\to D} y^*(x) = 0$$

by the strict concavity of $y^*(x)$ it follows that:

$$y^*(x) = y^*(D+(x-D)) < \frac{1}{1-\theta_y}\frac{\beta_x(D)}{\eta_x+\beta_x(D)}(D-x)$$

for each $x \in [0, D)$. Thus, the function $y^*(x)$ lies below the straight line:

$$y = \frac{D}{(1+\frac{\eta_x}{\beta_x^M})(1-\theta_y)} - \frac{1}{(1+\frac{\eta_x}{\beta_x^M})(1-\theta_y)}x \tag{16}$$

where $\beta_x^M = \beta_x(D)$. This line is the isocline of x in the model with a constant propagation coefficient, calculated at the maximum attainable PS propagation coefficient (Equation (7)).

Moreover, by assuming $\lim_{x\to 0} \beta_x(x) = 0$, we have:

$$\lim_{x\to 0} \frac{dy^*}{dx} = \frac{1}{1-\theta_y}\frac{\beta_x'(0)}{\eta_x}D > 0$$

and there is therefore a unique local maximum of the function ($y_M^* \in [0, \frac{D}{1-\theta_y}]$) lying in the domain $(0, D)$.[6] Hence, the function is as in Figure 3(a) and x will increase (decrease) depending on the combination (x, y) being actually below (above) the function.

What remains to analyze are the effects of the two parameters (θ_y and η_x) on the shape of the function. As for θ_y, it is sufficient to note that it enters the function simply as a multiplicative constant. Hence, an increase of θ_y moves x's isocline upward as in Figure 4(a), whereas a decrease of it moves the curve the other way around. As for η_x, an increase of it makes the curve change as in Figure 4(b) (see Appendix A for a more in depth analysis of the effects of the parameters on $y^*(x)$).

What about the isocline of y? Given that, also in the light of the analogy of assumptions (1)-(2) and (3)-(4), Equations (12) and (13) are symmetric with respect to the axes, the isocline of y is as showed in Figure 3(b). What we have said with respect to the isocline of x is therefore valid also for the other isocline provided that x and y are switched.

4 Conclusions

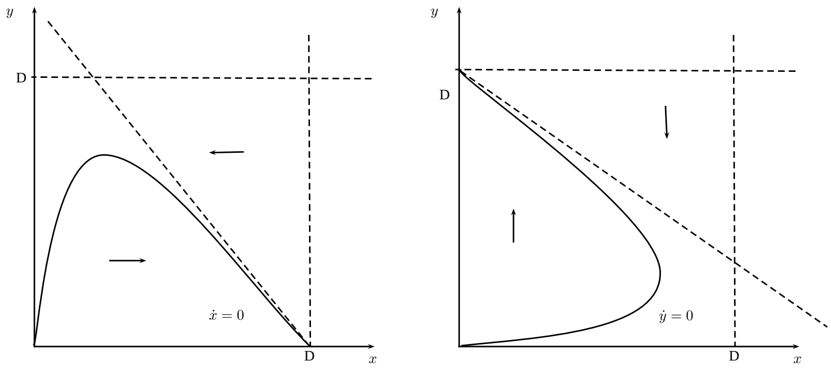
Our work has shown that the implementation of an innovation diffusion model is a useful formal tool in the task of modeling two competing technologies in high-tech industries. Indeed, being the latter ones characterised by the presence of economies of scale, network effects and the possibility of joint adoption, they need a proper theoretical investigation which, to our knowledge, has not been extensively carried out by the economics literature so far.

[6] When $\eta_x = 0$ we have:

$$\lim_{x\to 0} y^*(x) = \lim_{x\to 0} \frac{\beta_x(x)D}{(1-\theta_y)\beta_x(x)} = \lim_{x\to 0} \frac{\beta_x'(x)D}{(1-\theta_y)\beta_x'(x)} = \frac{D}{(1-\theta_y)}.$$

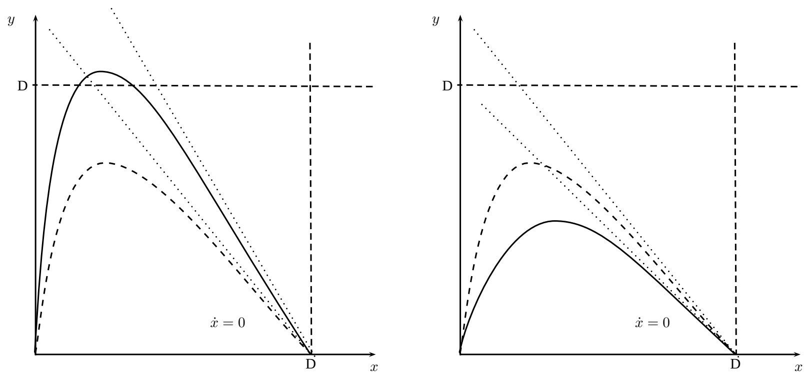
$$\lim_{x\to 0} \frac{dy^*}{dx} = \lim_{x\to 0} -\frac{\beta_x(x)^2}{(1-\theta_y)\beta_x(x)^2} = \lim_{x\to 0} -\frac{\beta_x'(x)\left(\beta_x'(x)+\beta_x''(x)\right)}{(1-\theta_y)\beta_x'(x)\left(\beta_x'(x)+\beta_x''(x)\right)} = -\frac{1}{1-\theta_y}$$

and we are back in the case analyzed in Section 3.2.1.



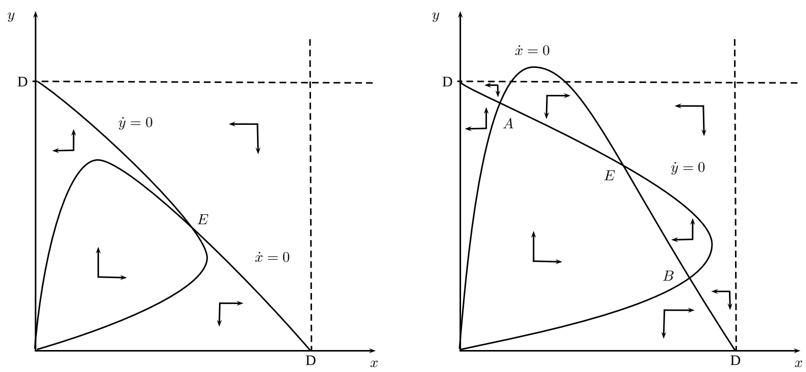
(a) Isocline of x (b) Isocline of y

Figure 3: Isoclines



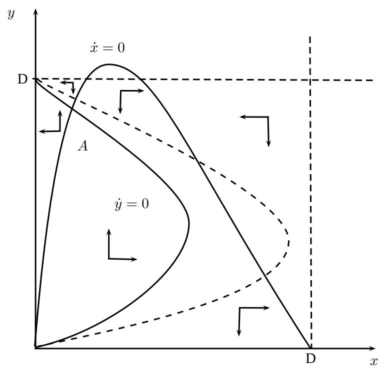
(a) Increase of θ_y (b) Increase of η_x

Figure 4: Changes of the parameters

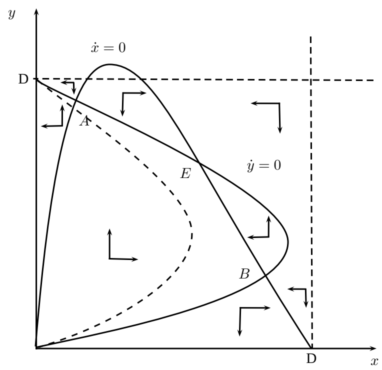


(a) Saddle point (b) Stable node

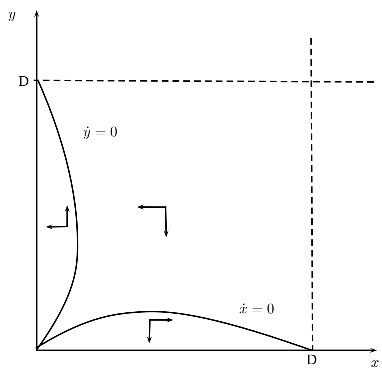
Figure 5: Rest points



(a) Decrease of OSS interoperability



(b) Increase of the probability of instantaneous joint use of OSS by PS users



(c) Strong lack of interoperability

Figure 6: Comparative dynamics

Moreover, if open source is taken into consideration, other issues come out, such as the effort of the community, developers' motivations and the degree of interoperability. All of these topics have been incorporated in a modified version of a standard epidemic model, which results as innovative to some respects. Above all, the endogenisation of the coefficient of propagation add complexity to the model general structure, yielding some interesting results. Among the others, it is worth stressing the fact that, contrary to standard results coming from the literature modeling the competition between OSS and PS, when the point of departures is highly unequal situation of lock-in one of the two technologies are likely to occur. In addition, comparative dynamics concerning the effects of the factors at stake have been presented.

A Effects of parameter changes on the isocline

As for θ_y, by the envelope theorem, the marginal effect of an increase of it on the maximum of $y^*(x)$ (y^*_M) is given by:

$$\frac{\partial y^*_M}{\partial \theta_y} = \frac{y^*_M}{1-\theta_y}(>0)$$

This marginal effect is therefore directly proportional to the initial level of the maximum and it increases for increasing values of the parameter ($\frac{\partial^2 y^*_M}{\partial \theta_y^2} > 0$).

As for η_x, its marginal effect is:

$$\frac{\partial y^*_M}{\partial \eta_x} = -\frac{y^*_M}{\eta_x + \beta_x(y^{*-1}(y^*_M))}(<0)$$

Also this effect is directly proportional to the initial level of the maximum and it increases for increasing values of the parameter ($\frac{\partial^2 y^*_M}{\partial \eta_x^2} < 0$).

Moreover, a change of η_x makes also the value of x corresponding to y^*_M change. In particular, an increase of η_x makes $y^{*-1}(y^*_M)$ decrease if and only if:

$$\beta_x(y^{*-1}(y^*_M)) > \beta'_x(y^{*-1}(y^*_M))(D - y^{*-1}(y^*_M)). \tag{17}$$

Indeed, from Equation (14) it follows that the FOC are satisfied if the expression in brackets is equal to zero. Working out the total differential of such expression and equating it to zero, after some algebraic manipulation we obtain:

$$\frac{dx}{d\eta_x} = \frac{\beta_x(x) - \beta'_x(x)(D-x)}{\beta''_x(x)\eta_x(D-x) - 2\beta'_x(x)(\eta_x + \beta_x(x))}.$$

The denominator of such expression is negative. Therefore it is negative if and only if condition (17) holds.

B Proof of the asymptotical stability of the equilibrium

B.1 Perfect interoperability

In order to prove the stability of the equilibrium in the case of perfect interoperability analyzed in Section 3.2.1, let us work out the Jacobian of the system at the equilibrium:

$$\mathbf{J} = \begin{bmatrix} -\beta_x(x^*)x^* & -(1-\theta_y)\beta_x(x^*)x^* \\ -(1-\theta_x)\beta_y(y^*)y^* & -\beta_y(y^*)y^* \end{bmatrix}$$

The discriminant of the associated characteristic equation is:

$$\begin{aligned} \Delta &= (-\beta^*_x x^* - \beta^*_y y^*)^2 - 4\left((1-(1-\theta_x)(1-\theta_y))\,\beta^*_x x^* \beta^*_y y^*\right) = \\ &= (\beta^*_x x^* - \beta^*_y y^*)^2 + 4(1-\theta_x)(1-\theta_y)\beta^*_x x^* \beta^*_y y^* > 0 \end{aligned}$$

The determinant of $\mathbf{J}$ is positive whereas its trace is negative, therefore both the eigenvalues are real and negative and (x^*, y^*) is a stable node.[7]

B.2 Non perfect interoperability

In the more general case of non perfect interoperability (Section 3.2.2), the Jacobian calculated at the fixed points is:

$$\mathbf{J} = \begin{bmatrix} -\beta_x(x^*)x^* & -(\eta_x + \beta_x(x^*))(1-\theta_y)x^* \\ -(\eta_y + \beta_y(y^*))(1-\theta_x)y^* & -\beta_y(y^*)y^* \end{bmatrix}$$

The discriminant of the associated characteristic equation is thus:

$$\begin{aligned} \Delta &= (\beta_x^* x^* + \beta_y^* y^*)^2 - 4\left(\beta_x^* x^* \beta_y^* y^* - (\eta_x + \beta_x^*)(\eta_y + \beta_y^*)(1-\theta_x)(1-\theta_y)x^* y^*\right) = \\ &= (\beta_x^* x^* - \beta_y^* y^*)^2 + 4(\eta_x + \beta_x^*)(\eta_y + \beta_y^*)(1-\theta_x)(1-\theta_y)x^* y^* (> 0) \end{aligned}$$

This discriminant is always positive, whereas the trace of the Jacobian is negative. Thus, the fixed points can be either saddle points or stable nodes depending on the determinant of the Jacobian being positive or negative. This determinant is equal to:

$$|\mathbf{J}| = \beta_x^* x^* \beta_y^* y^* - (\eta_x + \beta_x^*)(\eta_y + \beta_y^*)(1-\theta_x)(1-\theta_y)x^* y^*$$

and it is positive if and only if:

$$(1-\theta_x)(1-\theta_y)(1 + \frac{\eta_x}{\beta_x^*} + \frac{\eta_y}{\beta_y^*} + \frac{\eta_x}{\beta_x^*}\frac{\eta_y}{\beta_y^*}) < 1$$

that is, if:

$$-\frac{1}{(1-\theta_y)(1+\frac{\eta_x}{\beta_x^*})} < -(1-\theta_x)(1+\frac{\eta_y}{\beta_y^*}) \tag{18}$$

Inequality (18) is satisfied in point E of Figure 5(b). Indeed, in such point we have:

$$\frac{dy^*}{dx} < \frac{1}{\frac{dx^*}{dy}}$$

Given that, from Equation (14) it follows that:

$$\frac{dy^*}{dx} > -\frac{1}{(1-\theta_y)(1+\frac{\eta_x}{\beta_x^*})}$$

Recalling the symmetry between Equation (12) and (13), we have:

$$-\frac{1}{(1-\theta_y)(1+\frac{\eta_x}{\beta_x^*})} < \frac{dy^*}{dx} < \frac{1}{\frac{dx^*}{dy}} < -(1-\theta_x)(1+\frac{\eta_y}{\beta_y^*})$$

and the point E is therefore a stable node.

[7] For a systematic treatment of the methods of analysis of systems of non linear differential equations see, for instance, Medio and Lines (2001).

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