Structural Change of Production and Consumption: 
A Micro to Macro Approach to Growth

Tommaso Ciarli*, André Lorentz†, Maria Savona‡, Marco Valente§

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Abstract

The primary aim of the paper is to analyse the relation between structural changes in the organisation and composition of production, changes in income distribution, and the evolution of consumption, as affecting patterns of economic growth. Our kernel contribution is theoretical. We provide an (agent–based) micro–foundation to the link between structural change and growth by accounting for (i) firm–level organisational and technological changes, (ii) their impact on the structure of earnings and income of workers–consumers, and (iii) the consequent changes in consumption patterns. The model articulates the links between innovation and production on the supply side, and the endogenous evolution of income distribution and consumption ‘needs’ on the demand side. We let these links interact to identify, via numerical simulations, different scenarios of changes in the composition of economies, and their aggregate growth, as emerging properties of evolutionary micro–dynamics of innovation, skill and functional composition of employment, income distribution and consumption patterns.

Keywords: Structural change; consumption; earnings distribution; growth
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*CIBI, Manchester Metropolitan University, UK, tommaso.ciarli@unibo.it
†Max Planck Institute of Economics, Jena, Germany, lorentz@econ.mpg.de
‡University of Lille 1, France and University of Cambridge, UK, Maria.Savona@univ-lille1.fr
§University of L’Aquila, Italy, valente@ec.univaq.it
1 Introduction

This paper investigates the relation between structural changes in the organisation and composition of production, changes in income distribution, and the evolution of consumption, as affecting patterns of economic growth. Although these dynamics are strongly inter-linked, few contributions have systematically investigated their co-evolution, both in theoretical and applied literature. Even more so, the analysis of the micro-to-macro mechanisms behind these processes has been greatly overlooked by both mainstream and non-mainstream literature.

The ambition of this work is therefore to provide (agent-based) micro-foundation to the link between structural change and growth by accounting for firm-level organisational and technological changes, their impact on the structure of earnings and income of workers-consumers and the consequent changes in consumption patterns. We propose a model which articulates the links between innovation and production on the supply side and the endogenous evolution of income distribution and consumption 'needs' on the demand side. We let these links interact to identify via numerical simulation different scenarios of changes in the production composition of economies and aggregate growth as emerging properties of evolutionary micro-dynamics of innovation, skill and functional composition of employment, income distribution and consumption patterns.

This work adds therefore to the literature on growth and structural change in two main respects. First, from a theoretical point of view, we embrace structural change of production and consumption in the belief that both should be accounted for in any attempt to explain growth dynamics, in line with the classics of Pasinetti (1981). Second, from a methodological point of view, we do so by carefully crafting firms and consumers micro-behaviours and identifying the resulting macro-level scenarios of structural change and growth.

A second intended, and much needed, contribution of this work with respect to the existing literature is the explicit introduction of income distribution as one of the main channels between changes in the organisation of firms and production structure on the one hand and changes in the consumption patterns on the other one. We do so in three main respects. First, we model an explicit relation between technological change and the organisation of production, which goes beyond the well-known skill bias effect,
in determining the distribution of income (via earnings and profits share). Second, we suggest and model the relation between changes in income distribution and changes in consumption. Third, by endogenising the role of income distribution we are able to provide a valuable tool to extend the use of the model to policy simulations and derive normative implications.

The remainder of the paper is organised as follows. Next section locates the contribution of this work within the context of a selected sketch of both theoretical and empirical, firm– and macro–level streams of literature relevant to the mechanisms explained by the model. Section 3 describes the components and the functioning of the model, which is formally detailed at great length in the Appendix. Section 4 provides the preliminary results based on the numerical simulation of the model in the case of two simple scenarios. Finally, Section 5 summarises the rationale behind the model, discusses the preliminary results and, most importantly, proposes few lines of research which the various extensions of the model might usefully contribute to.

2 Background

There is still a considerable hiatus between what economic theory is able to explain and what actually happens in economic reality. Streams of economic theories still clash with respect to the range of phenomena investigated, the formulation of the main hypotheses, the justification of these latter, and, finally, the methodology employed. The present section reviews the relevant pieces of literature and the empirical stylised facts which support the choice of our assumptions as well as the selection of the main mechanisms in the model described in section 3 and formalised in Appendix. In what follows we attempt therefore not to enter in the never–ending (and so far sterile) debate between mainstream (i.e. neo–classical) and unorthodox scholars when searching for – and formalising – the ultimate determinants of countries’ different patterns of economic growth. Cross–country divergence in growth rates has been a solid empirical stylised fact for decades (Denison, 1967; Denison, 1979; Maddison, 1987; Barro, 1991). What is left is to assess to what extent the (change in the) sectoral composition of economies is responsible for it and, ultimately, what determines changes in the production structure.

Technical change, changes in the production structure and the evolution of demand might disrupt the sectoral composition of the economy (Pasinetti,
1981) and the steady path of macroeconomic growth. Very few scholars have attempted to look at both the supply– and demand–side as determinants of growth and structural change (Verspagen, 1993; Verspagen, 2004; Montobio, 2002; Llerena and Lorentz, 2004; Lorentz and Savona, 2006; Ciarli and Valente, 2005; Ciarli, 2005). Each of these contributions proposes models of economic growth which encompass both technical change and demand. However, none of them attempt to specifically look at the interaction between structural changes in the production and organisation of firms and structural changes of consumption needs to derive results on how changes in the composition of the economy affect aggregate growth.

Recent contributions (Saviotti and Pyka, 2004; Saviotti and Pyka, 2006) have looked at economic growth driven by structural change of the economy, in particular as a result of the emergence of new sectors, a phenomenon which is labelled interchangeably as the creation of product variety. Despite being greatly welcome as one of the very few attempts to focus on this issue, we still feel uncomfortable with the representation of structural change as limited to an increased product variety, with no explicit reference to whether and to what extent the evolution of consumption ‘needs’ and firms’ effort to invent and innovate do interact in producing novelty.

A much needed stream of evolutionary literature is developing around the issue of how consumption ‘needs’ evolve, drawing upon evidence and theory derived from psychology (Valente, 1999; Witt, 2001; Witt, 2006; Babutsidze, 2007). The importance of these contributions is two-fold. First and foremost on a theoretical level, insofar demand is looked at through the lenses of consumer behaviour and psychological drivers. Further, on a methodological level, as an explicit micro–foundation of consumption theory is proposed. Yet, demand both constraints is constrained by the supply’s response to it. Changes in the structure of demand, however driven by psychological incentives, finds its natural interlocutor in whether and how firms respond to it – i.e. to what extent consumption ‘needs’ are met by the invention and successful commercialisation of new product on the market – as Schumpeter had already emphasised long time ago (Schumpeter, 1934).

To our knowledge, no one single contribution has explicitly disentangled at the micro–level the role of distributional changes as the natural channel of the evolution of consumption ‘needs’ into the evolution of actual demand, i.e. changes in the signals which firms receive from the market and which they
respond and adapt to. Rather, the large and consolidated literature on the two-sided link between economic growth and income distributional change remains confined to macro–level analyses, since the seminal Kuznet’s curve and the works by (Stiglitz, 1969; Tinbergen, 1975), greatly enriched later on (Atkinson, 1997; Aghion, Caroli, and García-Peñalosa, 1999; Aghion, 2002; Galbraith, Lu, and Darity, 1999; Galbraith, 1999).

As for the macro–evidence on the two–sided relation between growth and distributional changes (i.e. increase in income inequality) Aghion, Caroli, and García-Peñalosa (1999) presents an extensive review on this issue. The authors look at both wealth and wages inequality and provide evidence and theoretical support looking at three competing explanations of wage inequality: i) trade (and especially import of intermediate goods from developing countries); ii) skill bias (which seems to prevail over trade, although there is also evidence of large inequality within educational classes), considering both disembodied (see also Aghion, 2002) and embodied technical change. Within homogeneous educational classes, sources of inequality are attributed to learning and inter–sectoral mobility; and iii) changes in firms organisation (and skill experience), though not further defined. In line with Tinbergen (1975), wage dynamics and inequality are argued to follow the competing game between demand and supply of skills. Up to the 70s skills supply has increased more than demand pushing down the relative wages. In the following period the demand for skills has increased and so have done the relative wages. The authors assume though that the only force driving the demand for skills is technical change.

A different view is proposed by Galbraith, Lu, and Darity (1999) and Galbraith (1999), according to whom inequality in income, and in earnings, is due to the country economic structure. The Keynesian approach hints at the Kuznets hypothesis and the specialisation effect on a global market. In arguing his point, Galbraith criticises the overrated explanation of wage inequalities led by skills. Wage distribution ultimately depends on the specialisation of the economy, both at the international level (Prebish–Singer

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1Surprisingly, the role of distributional changes is greatly overlooked within the evolutionary stream of literature.

2According to Galbraith it is not the use of the computer tout court which is responsible for wage increases. Rather, it is the working condition in which it is used, which makes an entirely new working class to emerge (computers should make jobs easier). The difference is not between users and non–users of technology, but between users and producers.
hypothesis) and at the national level (à la Kaldor).

Conversely, a great deal of micro–level literature has looked at (changes in) firms’ size and organisational structure as affecting the (skill and organisational) composition of workers and executives and the wages structure, since the seminal contributions by Simon (1957), Lydall (1959), Rosen (1982) and further extended (among others, Waldman, 1984; Abowd and Margolis, 1999; Prescott, 2003). Some recent contributions have reprised the issue of the relation between the employer size and the effects on wage distribution (Brown and Medoff, 1989), (for a review, see Criscuolo, 2000), as well as the sectoral specificities of skill–biased technical change (Haskel and Slaughter, 2002).

The key–words of this stream of contributions are therefore firm size, number of and complexity of organisational layers internal to the enterprise, the proportion of executives and workers and the structure of pay (and wage premiums). The interesting feature of this literature (see for instance Prescott, 2003) is that, in line with what Galbraith suggests at the macro–level of analysis, the role of skills differentials is over–emphasised with respect to other factors, in determining earnings and income inequality. For instance, Caroli and Van Reenen (2001) suggest that there is a dynamic other than skill–bias technical change which depends on the organisational change and affects wage and earning distributional change. Namely, an increased decentralisation of production and work organisation demands higher responsibility to a single executive and an increase in wage compensation of executives follows. Technical change, especially ICT–related, requires complementary organisation change in order to be effective, therefore increasing demand for higher shares of higher skilled executives.

This stream of literature, however, at the cross–road between economics and management, excludes from the domain of analysis the impact of changes in the wage structure on the evolution of consumption, both in terms of (average) disposable income and of preferences.

Our conjecture, reprised at greater length in section 3, is that changes in the economic structure and (trade and sectoral) specialisation have been accompanied by changes in the organisational structure of firms and both have brought about changes in the wage and earning structure. Both micro– and macro–level mechanisms are therefore behind changes in the consumption patterns, which in turn affect changes of the production structure both at
the firm and sectoral level. We turn to the identification and representation of these mechanisms in section 3 and refer to A for their formalisation.

3 The Model

This section provides a stylised description of the main mechanisms and assumptions behind the model, which is formally detailed in the A.

Following existing Schumpeterian growth models (see, among others Chiaromonte and Dosi, 1993; Silverberg and Verspagen, 1995) we consider economies composed of a consumers sector and a capital sector. Unlike the existing literature, we account for the whole set of innovation strategies for a firm: process, product and organisational innovation. Such extension allows to endogenising a number of mechanisms that are shown in the empirical literature to be responsible for the skewed (Pareto) distribution of incomes (e.g. economics of superstars, profits sharing, and supply of skilled labour, in line with the work of (Lydall, 1959; Rosen, 1982; Rosen, 1981; Prescott, 2003) and in turn affect consumption patterns.

The micro–dynamics of production, innovation and consumption patterns rely on the product defined as a vector of characteristics, which satisfies users’ needs. This draws upon the work by Ciarli and Valente (2005) and is in line with the Lancasterian (Gorman, 1959; Lancaster, 1966b; Lancaster, 1966a) and post–Lancasterian approach to consumer theory (Lancaster, 1966b; Saviotti and Metcalfe, 1984; Gallouj and Weinstein, 1997). We draw upon this stream of literature in the belief that it has great potentiality of developing a theory of consumption which does not need to rely on non–realistic assumptions regarding homogeneity of both consumers and preferences. Further, extensions of the characteristics–based approach allow to reformulate and provide theoretical generalisation to the definition of innovation.

First, changes in the production processes are modelled as investment in different capital vintages. By changing vintage, firms alter the capital/labour ratio of their technology, affecting the composition of the labour market and the income distribution in the consumers market. Firms be-

\[^3\]This latter has so far suffered from the dominance of empirical approaches, in turn driven by the necessity of quantifying the intensity of innovation. However, this has been in our view detrimental to the theoretical refinements of the seminal Schumpeterian taxonomy of innovation typology.
longing to the final and capital good sectors make use of labour force with a (continuum) different degree of skills. Workers enjoy therefore different earnings, due to differentiated wages and dividends, have different consumption preferences, and in turn compose different consumer classes. We also model the vertical relation between buyers and suppliers.

Second, product changes are considered as a two-fold outcome of changes in consumers preferences and budget constraints on the one hand, and firms’ technological competition to acquire and increase shares of the market on the other hand. Product changes - i.e. product innovation - are defined by a limited set of radical and incremental changes in the vector of characteristics and needs defining the product itself. The creation of a new product (both final and intermediate) is pulled by the occurrence of a new need (demand) and defined in terms of a different vector of characteristics.

Third, organisational changes are defined in a two-fold way. First, within the firm, organisational change is defined as an alteration of the enterprise’s governance structure, which affects the relative economic importance of executives and in turn income distribution and consumption behaviour. The latter is in fact linked to preferences formation within working classes and imitation across classes. Second, across firms, organisational change occur in terms of outsourcing. This latter phenomenon represents, from a product–characteristics point of view, the creation of a new (intermediate) product. The occurrence of new products on the (final and intermediate) market represents the very essence of structural change in the production, which is both affected and affects the structural change in income distribution and in (final and intermediate) consumption.

Organisational change as defined above as well as the emergence of new products are the main responsible for structural changes in earning and income distribution. As mentioned in the previous section, we rely on the seminal contributions by Simon (1957), Lydall (1959), Rosen (1982) and further extended (among others, Waldman, 1984; Abowd and Margolis, 1999; Prescott, 2003). In this context we aim to extend this literature by linking it to the production and innovation processes on the one hand and the consumption patterns on the other one. Our conjecture is that the changing weight of different type of workers within the firms’ governance structure as well as the changing weight of most complex governance structures within the whole distribution of firms on the one hand and the
emergence of new firms on the other one determine changes in the structure of earnings and income. Changing classes of income in turn affect the composition of consumption classes, which, according to the mechanisms stylised above, modify firms’ expectations in terms of market shares and, accordingly, constraint their actual production plans.

Finally, consumption behaviour draws on both economic and psychological evidence collected from marketing studies, and adapts the theoretical construction developed by Valente (1999) in earlier work. Demand comes from a number of consumer and therefore income classes. The composition of each class is defined in terms of a distribution of ‘needs’, from basic to luxury goods. Further, the distribution of consumers’ preferences over the goods’ characteristics defines the demand curve and firms’ production shares. Price enters as a threshold characteristic, given the budget constraint characterising each consumer’s class.

The description of the model is organised in different sections. These relate firstly to the supply side, in turn organised in terms of final (Section 3.1) and capital goods sectors (3.2) and secondly to the demand side, which includes income distribution and consumption patterns (Section 3.3). Once defined the final and capital goods, each section in the supply part is further organised in terms of the main characteristics and dynamics of firms’ behaviour. Among these, the production function itself, the innovation strategies, the wage and price setting, given the determinants of the minimum wage at the macro–level. Further, within the the demand side section, the dynamic of consumer’s choice as a function – among others – of budget constraints and preferences characterising the different income classes is detailed.

3.1 Final Good Firms

3.1.1 Product definition

Drawing upon the work by Ciarli and Valente (2005), which is in line with and extends the Lancasterian (Gorman, 1959; Lancaster, 1966a; Lancaster, 1966b) and post–Lancasterian approach to consumer theory (Saviotti and Metcalfe, 1984; Gallouj and Weinstein, 1997), the product is defined as a vector of characteristics which satisfy a set of one or more consumer needs. Each characteristics is assigned a given level of quality. We refer to section A.1.1 for a formalisation of it.
The definition of a product as a vector of characteristics crucially allows, according to us and in line with the post–Lancasterian literature, to minimise the arbitrariness of relying on hypotheses such as product homogeneity and of defining innovation in terms of an occurrence of a new product or an incremental change of it. Further, this theoretical framework is consistently employed below (3.1.7) in order to define product innovation with different degrees of radicality.

3.1.2 Production process

Each firm produces one product defined as above 3.1.1. Firms set up their production plans at the beginning of the period. Decisions on the actual volume of production are taken on the basis of the expected orders expressed by consumers, according to an adaptive decision rule (i.e. on the actual level of sales from the previous period 2). It is therefore the actual demand which determines the volume of production. This is in line with most of the recent literature aiming at embracing Keynesian features within neo–Schumpeterian growth models, (Verspagen, 2000; Verspagen, 2002; Landesmann and Stehrer, 2006) as well as the post–Keynesian stream of research (see, among others Thirlwall, 2002).

A mechanism of stock management is also included, such that it allows firms to cover orders which exceed the level of production at the given period. Should these orders be exceeding the total amount of current production and previous stock, they remain as backlogs and satisfied in the following period, once the firm has adapted the volume of production to the amount of orders collected. The actual level of stocks is therefore given by the sum of the stocks inherited from the previous period, the current production level minus the amount of backlogs collected.

The actual level of production is therefore a function of production plans – adjusted from period to period to the volume of demand – as well as of inputs constraints. The production function in fact also includes the effects of process innovations aiming at increasing labour productivity embodied in the capital vintages, as well as through investments in R&D carried out by the capital suppliers (see 3.1.6 and A.2.3). The level of production is constrained by the available labour capacity corresponding to the number of workers employed and hired at the previous period and the labour productivity embodied in the machinery available 6. The formalisation of the
production function, as well as the mechanism of stock management, are reported in A.1.2.

### 3.1.3 Labour

As mentioned before, a great deal of micro-level literature has looked at (changes in) firms’ size and organisational structure as affecting the (skill and organisational) composition of workers and executives and the wages structure, since the seminal contributions by Simon (1957), Lydall (1959), Rosen (1982) and further extended (among others, Waldman, 1984; Abowd and Margolis, 1999; Prescott, 2003).

According to this stream of empirical literature, firm size, number of and complexity of organisational layers internal to the enterprise – i.e. the proportion of executives and workers – affect the structure of pay (and wage premiums). We revert to this stream of literature to formalise the structure and composition of the workforce within the firm. As formalised in A.1.3 the composition of the workforce within a firms is mainly a function of the number of layers. Production process requires labour input, represented by the number of workers in each hierarchical tier. Further, firms employ a given number of (different tiers) executives whose role is exclusively to coordinate the workers in each tier.

This allows us to link the structure of earnings to firms’ organisational structure. It is worth noting that we avoid to confine the relationship between structure of earnings and organisation of the firm to the mere differences in the skill content of labour. The distinction between workers at different layers and executives does not overlap with the different degree of skilled labour. As a consequence, changes in the size and the organisational structure of the enterprise, regardless of the changes in the skill intensity of the workforce employed, are responsible for changes in the structure of wages and earnings.

### 3.1.4 Wage and price setting

Prices are set at the firm level. Firms set prices on the basis of:

- the minimum wage level (determined at the macro–level) (see sections 3.1.5 and A.2.4)
• the number of tiers and executives (i.e. the organisational structure of the firm) (see sections 3.1.3 and A.1.3)

• the wage premium paid to the executives, which is in turn a function of the financial resources made available by the cumulated profits \(^4\)

• the mark–up ratio applied to variable costs (average wage)

It is worth noting that for simplicity, we do not assume the presence of economies of scale, so that unit production costs depend on the average wage (variable costs). However, it should actually be noted that there are dis–economies of scales, which depend on the wage scale.

Two issues should be raised here. First, the fact that labour cost is higher for large firm is quite a robust evidence, as it is shown for example in Criscuolo (2000), Idson and Oi (1999), Bottazzi and Grazzi (2007). Second, a compensation mechanism occur (which would be interesting to analyse in future extension of the model) due to investment in capital: as firms grow, they hire more workers, increase the complexity of the organisational structure in terms of layers, therefore increasing the production costs. However, they also invest in new capital vintages, which leads them to increase the productivity, so that economies of scale actually emerge dynamically, rather than being assumed. This is in line with some neo–Schumpeterian models (Verspagen, 1993; Llerena and Lorentz, 2004) which have reprised and provided micro–foundation to the Kaldor–Verdoorn Cumulative Causation mechanism (Kaldor, 1957; Verdoorn, 1949).

A formalisation of the price determination at the firm level is provided in [A.1.4]

3.1.5 Minimum wage

The minimum wage is negotiated at the macro–economic level and defines the lowest bound of firms’ wage bargaining (section [A.1.4]). We assume that

\(^4\)The model does not include a financial market. However, a mechanism of profits redistribution is considered, such as the cumulated profits of the firm are firstly devoted to capital expenditures (see section [A.1.6]); the residual is then devoted, according to a firm–level decision rule, respectively to R&D to allow for product innovation and to bonus and dividends. It is worth noting that, since firms have no financial constraints, the decision made in any period \(t\) reduces the resources in any period \(t+n\), affecting the use of cumulated profits in \(t+n+1\).
the negotiation is linked to three main macroeconomic indicators:

- average labour productivity growth (to keep the pace of labour value contribution)
- average consumer prices (to hold the purchasing power)
- unemployment (as a continuous adjustment due, for example, to efficiency wages, corporatism, or bargaining)

On the one hand we allow the minimum wage to continuously adapt to changes in the labour market, following the ‘wage curve’ negative relation between wages and unemployment. The wage curve is in fact a well-established empirical evidence at the macroeconomic level (Blanchflower and Oswald, 2006; Nijkamp and Poot, 2005). On the other hand, the minimum wage is fully re-negotiated when the increase in both average price and aggregate productivity — with respect to the previous negotiation (at $t_0$) — exceeds a boundary level ($\Omega^P$ and $\Omega^A$ respectively). This second dynamic follows the discrete negotiations in the national contracts, which occurs after bargaining periods between ruling bodies and labour unions. The formalisation of the minimum wage formation is reported in Appendix A.2.4.

We revert to the empirical stylised facts which corroborate our assumptions related to the functional form of the minimum wage formation and support our choice also with respect to the assumptions on the functioning of the labour market. Given our earlier assumption of unconstrained labour resources — in order to avoid threshold solutions defined by the population level to determine the model’s results — we need to specify an alternative way to derive unemployment rates. We use another well-established empirical evidence (a stylised fact), the Beveridge Curves, that shows a negative relation between the rate of vacancies and the rate of unemployment. We assume that the labour market can be well represented by a matching model (e.g. Petrongolo and Pissarides, 2001; Yashiv, 2007). In other words, the higher the unemployment, the easier it is for firms to look for workers and match; and the higher the number of vacancies, the easier for workers to find a firm, and match. Further, we follow Fagiolo, Dosi, and Gabriele (2004) — who show that random matching models fail to reproduce Beveridge curves

\footnote{(For an extensive review of the empirical evidence, see for instance Cornia, 2003; Cornia and Rosignoli, 2006)}
— and assume path dependency in both labour supply (workers are loyal to firms) and demand (firms react cautiously to market signals). Again, the functional form of the Beveridge curve used in the model is formally detailed in A.2.4.

### 3.1.6 Capital and investment

Following Amendola and Gaffard (1998) and Llerena and Lorentz (2004), machinery or capital goods are not used as production factors, but constitute the basis for firm’s production capacity. The accumulation of capital is a pre-condition for any production activity, constraining the actual production level and affecting the efficiency of use of the labour force.

The accumulation of capital allows the firms to build their production capacity. This is a pre-condition to production. Investments in capital goods therefore allow firms to:

- increase the production capacity of the firm ($\bar{BK}_{t-1}$)
- increase the efficiency of the production process, increasing labour productivity due to the embodied nature of technical change ($A_t$)
- modify the structure of the work force employed due to the organisational changes required by the use of new capital goods

Firms invest in capital goods through the following decision rule: To keep their capital intensity ($\frac{1}{\bar{B}}$) fixed, firms invest as (i) the capital stock depreciate and as (ii) the expected sales increase.

Firms then place orders to the machinery firms (see 3.2 corresponding to their investment decision. More a detailed formalisation of firms’ decision to invest is provided in A.1.5.

### 3.1.7 Product innovation

### 3.2 Machinery Firms

#### 3.2.1 Definition of capital goods

Each capital goods is characterised by its vintage ($\tau$), an embodied level of productivity ($a_\tau$), the skills ratio ($\nu_\tau$) required to use it and a technology type ($\theta$). The technology type limits the number of final good firms able to use the capital goods produced. Only the firms using a technology of
the type $\theta$ are able to use capital goods of the same type (i.e. a producer of textile products cannot use the exact same technology as a computer producer). The characteristics embodied in the capital goods ($\tau$, $a_\tau$, and $\nu_\tau$) are an outcome of the R&D activity of the machinery firms.

### 3.2.2 Production process of capital goods

The total demand for capital goods at a given period $t$ ($K^d_t$) corresponds to the sum of the orders addressed to the machinery firm at the same period ($k^d_j,t$) and the orders uncovered by the production at the previous period ($U^d_{t-1}$). We assume production of capital is just–in–time, without any need for expectation formation or accumulation of stocks of unsold capital. First, the price of a unit of capital is quite costly to allow firms to retain an unsold amount of machinery. Second, the use of capital is quite firm specific, and the transaction results from an interaction between the supplier and the buyer, such as the type of capital which is sold. Moreover, differently from consumers demand, the demand for capital is much more stable through time. The actual production level ($Q^k_t$) of machinery firms is constrained by their production capacity ($\bar{A}^k_t L^k_t - 1$). The capital orders of the final good firms are treated in order of arrival and orders are never delivered if partially completed. Hence the actual production level of the $c$ can be defined as follows:

The total sales ($Y^k_t$) by a machinery firm for a given period therefore corresponds to the sum of the order completed during this period.

The production capacity of a machinery firm corresponds to its fixed labour productivity ($\bar{A}^k$) time the labour force available for production. The latter corresponds to the work force employed and hired at the previous period ($L^k_{t-1}$)

Machinery firms hire workers ($L^k_t$) to increase their production capacity to cover all their orders ($K^d_t$) and to keep a share ($u^m$) of unused workers to cover some unexpected demand.

### 3.2.3 Price setting and Profits

The price for capital goods $p^k_t$ are set using a mark-up ($\mu^k$) on the production and development costs of the machinery firm. The latter respectively corresponds to the wage bill for workers hired for production and engineers hired for R&D.
Wages paid to worker \((w^k_t)\) and to engineers \((w^E_t)\). These wages are set by firms applying a coefficient \((\omega^k\) and \(\omega^E\) respectively) to the minimum wage set at the macro-level \((w^{m}_{t-1})\).

The profits \((\pi^k_t)\) are then redistributed as dividends by the machinery firms to their shareholders \((Div^k_t)\).

### 3.2.4 R&D and Innovation in Machinery

Machinery firms aim to improve the characteristics of their products through their R&D Activity. The outcome of this activity is stochastic. The R&D activity is directly in line with Nelson and Winter (1982), and follows the scheme developed in Llerena and Lorentz (2004).

The probability \((p^{inn}_t)\) of success of R&D increases with the number of engineers employed in R&D \((L^E_{t-1})\). The R&D investment by machinery firms therefore consist in hiring engineers. We assume here that firms correlated the engineers pool to their profits in the range of a fixed share \(\rho\) of cumulated profits constrained by a fixed ratio \(\nu^k\) of engineers by workers \((L^k_t)\).

Once the R&D process is successful, the characteristics of the newly developed capital vintage are themselves stochastically defined. The embodied productivity level \((a^T)\) of the newly developed vintage corresponds to the value for the previous vintage \((a^{T}_{t-1})\) augmented from \(\varepsilon^a_t\), drawn from a Normal distribution with a fixed standard deviation \((\sigma^a)\), and a mean equal to zero. The improvement of capital goods is then uneven and marginal in time.

Similarly the skill ratio \((\nu^T)\) of the newly developed vintage, is drawn from a normal distribution centred on the previous vintage’s ratio with a fixed standard deviation \((\varepsilon^\nu_t)\). As the ratio grows the higher the skill requirements for the newly developed vintage, technical change is then skill biased. Reversely as the ratio decreases the lower requirements for skills. In this latter case, technical change substitute skills to machinery requiring less skills to produce in the final good sector.

### 3.3 Aggregate demand

Following the same approach used in the rest of the model we implement aggregate demand according to a micro-founded representation, devised to replicate empirical evidence at single consumer level. Such consumer is
however defined in a meso- and macro-context allowing to express static and dynamic properties deemed relevant in the concerned literature.

The demand module “closes” the income cycle with the supply side. On the one hand, income is generated in the supply side as wages and distributed profits. On the other hand, this same income is spent by households to purchase goods and services provided by the supply side. The aim of the demand module is to represent the mechanisms by which aggregate disposable income is turned into firms’ revenues.

The purchasing decisions of the individual consumer are represented as influenced by several factors, which the model allow to represent and we assume can be calibrated using data available from different sources, although, for analytical purposes, any of those factors can be ignored or simplified. In the following we briefly summarize the factors and the reason for their inclusion in the model. In the rest of the section we describe their technical implementation.

Firstly, a consumer is assigned a disposable income, the total amount of money she can spend with the unit of time considered. We apply the widely accepted hypothesis that consumers’ behaviour is driven by long-term expenditure capacity, and, therefore, short term fluctuations influence only marginally current consumer behaviour. Differentiating between currently received income and current consumer expenditure we leave also open the opportunity for including in the model other sources of income, as, for example, savings’ or financial investment income, foreign remittances, governmental payments, etc.

Secondly, consumers are assumed to influenced by social and cultural factors, influencing their preferences. A large literature, mostly applied, describes and measures ever evolving social classes with peculiar tastes, like, for example, yuppies, soccer mums, “Joe Sixpack”, etc. The nature and dimension of these classes is actively studied by the marketing literature given its capacity to explain and forecast current and future direction of consumption patterns.

Thirdly, we assume that social and income classes identify a distribution of needs, that is, classes of goods and services providing common functions to the consumers. We consider that any purchase satisfies a specific need, and that consumers (located in a certain income and social class) maintains approximately constant distribution of expenditure for each of the needs.
Lastly, we consider that potential purchases are perceived by consumers as defined by their cost (price) and by a vector of characteristics evaluating the good or service in respect of a given need. Such evaluation is partly objective, being a function of products’ or services’ objective characteristics, and partly subjective, varying with the tastes of consumers. Consumers aim, within their capacities, to choose, among available options, the purchases better fulfilling their different needs.

Let’s see, in detail, the implementation of the demand side of the market. We will start from the behaviour of the consumer, then will move to the classes of consumers and, finally, to the whole aggregate demand.

3.3.1 Micro-consumer behaviour: single consumer purchasing decision

A consumer considers purchases as possible solutions to satisfy a given need. Products and services are associated to a vector of characteristics, mapping the properties of the good or service onto one of the characteristics relevant for the need. This mapping should itself be influenced by relevance and role of the need within the social class of the consumer, so that, for example, fulfilling the same need “X” the same product scores differently for members of different social classes. However, as a first approximation, we replace this aspect by means of a stochastic function translating “objective” values of each product on a need (identical for all the classes of consumers) in “subjective” values, as perceived by the consumer. The function is a normal random variable centered on the objective value of the product and whose variance is an indicator (class-specific) of the capacity of the consumer to deal with that specific property of the product. For example, the variance concerning the property price is nil for all consumers, while the properties concerning, say, technical aspects of a complex product will be large for low education classes and decreasing for highly educated ones. The randomness of the values of a product for a consumer allow also to describe consumers’ variety within a given class.

The routine determining the purchases of a consumers considers initially as part of the potential option set all the goods and services currently available on the market, and proceeds by successively removing different options.

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6 An interesting extension consists in considering as the initial option set only offerings satisfying a given criterion, for example having high market shares or other forms of
Notice that, since consumers decide on the basis of stochastic values, the result will be itself stochastic. The model, as we will see, uses probabilities of purchases expressed as frequencies of repeated runs.

The first selection is performed on the basis of a set of “minimal requirements”, represented by a vector of minimum values that positive characteristics must provide in order to be part of the option set (or maximum values for negative characteristics). For example, consumers remove initially from the option set all products with too high prices. Note that, given the stochasticity of values, we may even (rarely) expect quite counterintuitive events, allowed by the long tails of the distribution.

Consumers’ preferences are represented as an ordering of the purchase characteristics. Such ordering determines the relevance of the dimensions defining a purchase, and is assumed to vary across different social classes and be identical within each class. Representing preferences as an ordering has several advantages. Firstly, it gives a sharp definition of a concept that, in most of the literature, is never identified, but only cited residually to justify any consumers’ behaviour not otherwise motivated. Secondly, the definition separates the actual decisions from the criterion used to reach the decision. Thirdly, the definition is highly compatible with most of the literature on market research discussing the actual decisional processes of real consumers and the measurable differences among them. Actually, a market research technique (conjoint analysis), collects the ordering of characteristics in order to perform statistically efficient market tests on new products, implicitly adopting the definition of preferences as an ordering.

The consumer decide which product to purchase in a given option set, adopts sequentially the characteristics as listed in her preferences. For each of them, the consumer computes the best option as measured by its (perceived) value on that characteristic. Then the consumer uses a percentage, called tolerance, expressing a threshold on the options’ values below which worse products are discarded. For example, suppose the best option scores 500 on the currently used characteristics (supposedly a positive one), and tolerance is 80%. Then, all options scoring 400 or less are removed from the option set, while the others remain, being assumed as equivalent to optimal option. The tolerance index is specific for each class and each user visibility. For example, aspiring consumers may consistently put in their initial option set all current purchases of upmarket social classes.
characteristic.

The routine filters away all options using the characteristics’ order as indicated in the consumer’s preferences. Using earlier the most relevant characteristics ensure that, stopping the selection, one still has the best (or equivalent, as defined by tolerance) options on that characteristics within the option set. The stopping can take place at any phase, representing different types of purchases, though, for the simulation tests performed so far, we allowed the selection to continue until at least an option remained in the option set, or all the characteristics had been used. Finally, within the option set remained after the selection terminates an option is chosen randomly.

Concluding, the decisional routine for a consumer requires the following data.

- Products defined over a vector of users’ characteristics. These value depend on the product own intrinsic properties and on the need the consumer is aimed to. These data concern products and services, measured against the possible uses for different needs.

- Vector of variances of stochastic error function, turning the values above into subjectively perceived ones. This vector and the following are all class specific.

- Vector of minimal requirements, to be used to remove at the outset irrelevant options.

- Ordered vector of preferences, representing users’ characteristics.

- Vector of tolerance levels, to be used to define a range of equivalence around the optimal values for each characteristic.

- Stopping rule (optional), determining how many characteristics are used in the purchase.

The routine described requires a quite large amount of information concerning consumers. However, all necessary data are available, at least approximately, or can be easily estimated from different sources. Also, the routine is able to generate a wide diversity of behaviours, each of them mapped on one or more set of parameters determining the consumers, allowing the modeller to tune the demand representing basically any consumer’s
profile deemed necessary. Therefore, one can either use this module either to search for specific parameterization generating known results, or, viceversa, observing the results generated by a specific parameterization.

This routine represents single-consumers behaviour. The following paragraph describes how it is used to represent aggregate demand behaviour.

### 3.3.2 Class demand

Classes are defined both in terms of income and social parameters. Sources for the definition of classes can be obtained by national accounts and surveys conducted by national statistical offices. Definition of social classes are regularly produced by public and private institutes. We assume that a social class is defined by the parameters defining the purchasing behaviour of its member consumers and, at each period, a total amount of expenditure, which a class demand routine distributes as sales across firms in the market.

The routine divides the total expenditures of the class to each need, according to shares defined in the class. For example, high-income classes will have a smaller shares of expenditure devoted to food and clothing, while low income classes will have a higher share for such needs. Next, for each need, expenditures are divided to firms, producing (summing up all needs for all classes), the period’s revenues of firms. Physical sales are obtained by dividing revenues of a firm by its unit price.

To allocate the expenditure of a class for a need to firms it is repeatedly used the consumer’s decisional routine. At each repetition new random values (for the perceived purchase qualities) are re-drawn, so that, in effect, the repetitions represent a sort of a sampling of the population of consumers in the class, with the variety represented stochastically. During each run of the consumers’ decisional routine the results are recorded as increasing a counter for the firms selected. At the end of all the repetitions the normalized values of these counters for each firm are used as shares to distribute the expenditures earmarked for the need.

### 3.3.3 Aggregate consumption

Summarizing the previous paragraphs, we have that classes distribute their income to different needs and, for each need, the expenditures are turned into revenues of firms, and eventually in sales, using a sample of consumers
in the class. In the following we describe how aggregate consumption is
determined and distributed to classes.

As we already noted we distinguish the income for households from wages
and other sources from the expenditures they make in a period of time. We
assume that consumers smooth their expenditures in respect of the more
fluctuating income dynamics, as if they were drawing from saving accounts
where the current income is deposited. The smoothing allows to maintain
medium and long term consistency between income and expenditures, avoid-
ing too strong feedbacks from between sales and income.

Different sources of income have different relevance for different classes,
who receive their total income according to shares from the different sources.
Each class then smooths the incomes to determine the actual available ex-
penditures.

3.3.4 Changes in demand

Depending on the research question targeted with the model it is possible to
implement changes to the parameters governing demand’s behaviour. For
example shares of income from different sources may be linked to demo-
graphic changes in the classes composition and consequently changes in the
income. Also, it is possible to consider changes in the consumers’ prefer-
cences and other behavioural parameters. These changes can be induced by
purposeful actions by firms, aiming at improving their profits, or passively
reflecting overall social changes. Among the possible changes it is also to
be considered the emergence of new needs, as novel types of consumptions
establish themselves and, typically, move from conspicuous to necessary.

4 Preliminary simulation results

To be completed

5 Summary of the findings and conclusions

To be completed
References


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A Mathematical Appendix

The appendix provides the formalisation of the model described in Section 3. For the sake of clarity of exposition, the appendix is organised symmetrically to Section 3.

A.1 Final Good Firms

A.1.1 Product definition

Each firm \( f \in [1; F] \) produces only one product. Each product satisfies one or more consumers’ needs \( n \in [1; N] \), defined over a vector of ‘use characteristics’ \( m_n \in [1; M_n] \) that quantifies the quality level of the service(s) they provide \( i_{n,m} \):

\[
\begin{pmatrix}
  i_{1,1} \\
  \vdots \\
  i_{n,1} \\
  \vdots \\
  i_{n,m} \\
  \vdots \\
  i_{n,M} \\
  \vdots \\
  i_{N,M}
\end{pmatrix}
\]

(1)

A.1.2 Production process and sales

Given the share of total demand faced by a firm \( Y_t \)\(^7\), current expected sales \( Y_t^e \) are a convex combination of past expectations and actual demand faced in the previous period:

\[
Y_t^e = a^e Y_{t-1}^e + (1 - a^e)Y_{t-1}
\]

(2)

In order to cover unexpected demand changes and meet the current orders from customers, firms maintain a desired level of stocks \( \bar{S} \). Production plans \( (Q_t^d) \) are then revised to adjust to changes in the expected demand.

\(^7\)Given that this section describes micro–behaviour, we suppress the firm index to improve readability. It is implicit that each equation is replicated for each firm, and we refer to the definition of parameters values to identify differences across firms.
(\(Y_t^e\)), actual stocks (\(S_t\)) and the orders that the firm could not fulfil in the previous periods (\(Bl_{t-1}\)):

\[
Q_{dt} = \max \{S_t - S_t^e + Y_t^e + Bl_{t-1}; 0\} \tag{3}
\]

where stocks and backlogs (failure to meet demand), are meant to work as production buffers, adjust in the following way:

\[
S_t = \max \{S_{t-1} + Q_t - Y_t - Bl_{t-1}; 0\} \tag{4}
\]

\[
Bl_t = -\min \{S_{t-1} + Q_t - Y_t - Bl_{t-1}; 0\} \tag{5}
\]

Backlogs and stock variations, a part from expectations failures, are due to input constraints in labour (\(A_{t-1}L_t\)) and capital (\(\bar{BK}_{t-1}\)). The current production is thus computed as follows:

\[
Q_t = \min \left\{Q_{dt}; A_{t-1}L_{t-1}; \bar{BK}_{t-1}\right\} \tag{6}
\]

where \(A_{t-1}\) is the labour productivity embodied in the capital vintages and \(\bar{B}\) is the capital intensity ratio. Both depend on firms investment in capital (see section A.1.5) and on the R&D of capital suppliers (section A.2.3). Both input markets are assumed to be inertial but unconstrained, where the available capital is limited by the suppliers production capacity (see section A.2.1).\footnote{We do not assume an infinitely elastic labour supply curve, as it will be more clear from section A.2.4}

**A.1.3 Labour**

Given the production plan, firms employ (displace) first tier workers (\(L_{t}^1\)) according to the overall labour productivity of the capital vintages at work (\(A_{t-1}\)), and in order to maintain an unused labour capacity (\(u_t^L\)) to cover unexpected demand:

\[
L_t^1 = \epsilon_L L_{t-1}^1 + (1 - \epsilon_L) \left[ (1 + u_t^L) \frac{1}{A_{t-1}} \min\{Q_{dt}; \bar{BK}_{t-1}\} \right] \tag{7}
\]

where the inertial factor \(\epsilon_L\) mimics labour market rigidities.

On top of first tier employees, firms need to hire an ‘executive’ to manage every batch of \(\nu\) first tier workers, and a third level executive for every group
of \( \nu \) second tier workers, and so on. The number of workers in each layer, given \( L_1^t \) is thus

\[
\begin{align*}
L_2^t &= L_1^t \nu^{-1} \\
L_3^t &= L_1^t \nu^{-2} \\
& \quad \vdots \\
L_\Lambda^t &= L_1^t \nu^{1-\Lambda}
\end{align*}
\]

where \( \Lambda \) is the total number of layers required to manage the firm, which can be obtained with a couple of algebraic transformations:

\[
L_1^t = L_1^{1+\Lambda-1}/\nu^{\Lambda-1} \\
\Lambda \ln \nu = \ln L_\Lambda^t + \ln \nu - \ln L_1^t \\
\Lambda \approx \frac{\ln L_\Lambda^t}{\ln \nu} + 1
\]

and the total number of workers is

\[
L_t = L_1^t + L_2^t + \ldots + L_\Lambda^t = \sum_{l=1}^{\Lambda} L_l^t.
\]

The relation between the number of first tier workers \( (L_1^t) \) and the number of layers \( (\Lambda) \) for different ratios of executive/subordinates \( (\nu) \) is depicted in figure ??.

### A.1.4 Wage, cost and price determination

First tier wages are set by firms as a multiple \( \omega \) of the minimum wage \( w_{l-1}^m \) defined at the macroeconomic level (see Section [A.2.4]):

\[
w_1^t = \omega w_{l-1}^m
\]

while higher level ‘executives’ earn a ‘relative’ wage which depend on the tier multiplier \( b \) (ratio between the salary of any executive and the salary of its immediate subordinates):

\[
\begin{align*}
w_2^t &= bw_1^t \\
w_3^t &= bw_2^t = b^2 w_1^t \\
& \quad \vdots \\
w_\Lambda^t &= b^\Lambda w_1^t
\end{align*}
\]

Executives at each level \( l \) are also paid a wage premium \( \psi_l \), when resources \( (R_l^D) \) from cumulated profits are available (see Eq. [20]). The premium is
proportional to the executives’ salary level:

$\psi^l = \frac{w^l_t}{\sum_{l=1}^L w^l_t} R^D_t$ \hspace{1cm} (13)

and overall earnings correspond to $w^l_t + \psi^l_t$ (where the second term is nil for first tier workers).

Assuming there are no economies of scale, unit production costs ($c_t$) then depend on the average wage ($\overline{w}_t$) (variable costs) for a given level of production capacity ($A_t$) (see Eq 22):

$c_t = \frac{\overline{w}_t}{A_t}$ \hspace{1cm} (14)

$\overline{w}_t = \frac{\sum_{l=1}^L w^l_t L^l_t}{L_t}$ \hspace{1cm} (15)

and the price is set as a simple mark-up ($\hat{\mu}$) over variable costs

$p_t = (1 + \hat{\mu}) c_{t-1}$ \hspace{1cm} (16)

Profits ($\pi_t$) then result as the difference between the value of sales and production variable costs of production

$\pi_t = p_{t-1} Y_t - \sum_{l=1}^L w^l_t L^l_t$ \hspace{1cm} (17)

Acknowledging that firms invest in capital when they face a production constraint, cumulated profits ($\Pi_t$) are eroded by capital expenditure ($R^I_t$)

$\Pi_t = \sum_t \pi - R^I_{t-1}$ \hspace{1cm} (18)

$R^I_t = \sum_{\tau=0}^t \rho^{k_{\tau-1}} \kappa_{\tau}$ \hspace{1cm} (19)

where $p^{k_{\tau-1}} \kappa_{\tau}$ are the resources used for capital vintages $\tau$ completed in time $t$.

The residual amount of cumulated profits is then allocated to product R&D ($R^{RD}_t$) and payment of bonuses and dividends ($R^D_t$), on a firm level decision rule

$R^{RD}_t = \rho \Pi_t$ if $\Pi_t > 0$ \hspace{1cm} (20)

$R^D_t = (1 - \rho) \Pi_t$

where $\rho$ is the share of profits devoted to R&D.
A.1.5 Capital and investment

The capital stock \((K_t)\) of a firm at the end of period \(t\) is given by the sum of the cumulated stocks \(k_\tau\) of capital vintages bought in periods \(\tau\), net of capital depreciation \((\delta)\):

\[
K_t = \sum_{\tau=0}^{t} k_\tau (1 - \delta)^{t-\tau}
\]  

(21)

And the productivity embodied in each vintage \((a_\tau)\) returns the maximum productivity of a firm, when all its capital is put into production:

\[
A_t = \sum_{\tau=0}^{t} \frac{k_\tau (1 - \delta)^{t-\tau}}{K_t} a_\tau
\]  

(22)

When the available capital stock does not allow to cover expected sales \((Y_t^e)\), the required investment in new capital units is defined:

\[
k_t^e = \left(1 + u^k\right) \frac{Y_t^e}{B} - K_{t-1}
\]  

(23)

where \(u^k\) is the ratio of unused capital to cover unexpected demand. An order of \(k_t^e\) units is then placed to a supplier \(f_h^k \in [1; F^K]\) that produces a compatible technology \(\theta\) among the producers in the capital sector. The \(h\) supplier is chosen with a given probability

\[
k_h = \left(\frac{p_{h,\tau-1}^k}{1 + p_{\tau-1}^k}\right)^{-\phi^p} \left(\frac{a_{h,\tau-1}}{1 + a_{\tau-1}}\right)^{-\phi^a} \left(\frac{u_{h,t}}{1 + u_t}\right)^{-\phi^u}
\]  

(24)

where \(\phi^p\), \(\phi^a\) and \(\phi^u\) are the buyer’s preferences with respect to the capital vintage’s price \((p_{h,\tau-1}^k)\), embodied productivity, and a proxy of the waiting time before the capital can be delivered \((u_{h,t})\) (the sum of the supplier’s standing orders \((U_{t}^d)\) weighted by the number of periods they have being cueing). Over–signed variables stand for non weighted averages across capital producing firms.

The actual number of capital units acquired in time \(t\) \((k_t^d)\) then depends on the production capacity of the capital supplier (see Section A.2.1):

\[
k_t^d = \begin{cases} 
  k_t^e & \text{if order} \leq \text{supplier production capacity} \\
  0 & \text{if order} > \text{supplier production capacity}
\end{cases}
\]  

(25)

and the firm may not acquire new capital before the previous order has been fulfilled.

33
A.2 Capital sector

Each capital good is characterised by its vintage $\tau$, productivity $a_\tau$, and a technology type $\theta$.

\[
\begin{pmatrix}
\tau \\
ad \\
\theta
\end{pmatrix}
\]

A.2.1 Production process of capital goods

Production plans for capital goods ($K^d_t$) aim to meet clients’ orders ($k^d_{j,t}$) and the demand uncovered in previous periods ($U^d_{t-1}$):

\[
K^d_t = \sum_{j=1}^{J_t} k^d_{j,t} + U^d_{t-1}
\]

where $j$ corresponds to a generic order arrived in time $t$, and $J_t$ to the last one (i.e. the number of orders).

As for the consumables sector, the production of a machinery firm is constrained by its production capacity ($\bar{A}^k_t L^k_{t-1}$).

\[
Q^k_t = \min \left\{ K^d_t; \bar{A}^k_t L^k_{t-1} \right\}
\]

The capital orders are treated on a ‘first in first out’ rule and they not delivered to clients unless completed. Therefore, $U^d_{t-1}$ is produced before any new order and total sales ($Y^k_t$) corresponds to the sum of the order completed ($k_{z,t}$):

\[
Y^k_t = \sum_{z=1}^{Z_t} k_{z,t}
\]

where $Z_t$ is the number of orders completed at time $t$, so that:

\[
\sum_{z=1}^{Z_t} k_{z,t} =
\begin{cases}
K^d_t & \text{if } K^d_t \leq \bar{A}^k_t L^k_{t-1} \\
U^d_{t-1} + \sum_{j=1}^{J_t} k^d_{j,t} & \text{if } K^d_t \geq \bar{A}^k_t L^k_{t-1}
\end{cases}
\]

for $k^d_{j,t}$ being the the last order that can be entirely fulfilled before capacity constraint. The orders remaining to cover in the following periods $U^d_{t}$ can be computed as follows:

\[
U^d_{t} = \sum_{j=J+1}^{J_t} k^d_{j,t}
\]
Notice that from equation 27 the production capacity is not left unused: a capital part may be built but not sold.

Given labour productivity \((\bar{A}^k)\), the production capacity then depends on the labour force employed \((L_{t}^k)\) to meet the production target \((K_{t}^d)\) while maintaining a share \((u^m)\) of unused workers to cover unexpected demand.

\[
L_t^k = \epsilon_M L_{t-1}^k + (1 - \epsilon_M) \left[ (1 + u^m) \frac{K_t^d}{A^k} \right] \quad (31)
\]

### A.2.2 Wage, costs and price determination

Assuming there are no economies of scale, unit production costs depend on the wages of workers, executives and engineers. The unitary price \((p_t^k)\) then includes both production and unit research costs, applying a mark–up rule \((\mu^k)\):

\[
p_t^k = (1 + \mu^k) \left( \frac{\overline{w}_{t-1}}{A^k} + \frac{w_t^E L_t^E}{A^k L_{t-1}^E} \right) \quad (32)
\]

where \(\overline{w}^k\) is the average salary throughout levels, computed as in the consumables sector (see equations 12 to 15), \(L_{t-1}^k\) the number of engineers involved in R&D. First tiers capital workers’ and engineers’ wage is also linked to the minimum wage, through a firm level bargain rule \((\omega^k\) and \(\omega^E\) respectively):

\[
w_t^k = \omega^k w_{t-1}^m \quad (33)
\]

\[
w_t^E = \omega^E w_{t-1}^m \quad (34)
\]

The profits \(\pi_t^k\) are either redistributed as dividends and bonuses or cumulated for future investment in engineers

\[
\pi_t^k = p_{t-1}^k Y_t^k - w_t^k L_{t-1}^k - w_t^E L_{t-1}^E \quad (35)
\]

### A.2.3 R&D and Innovation in Machinery Firms

The number of engineers employed \((L_{t-1}^E)\) positively influences the probability of a successful innovation in the capital productivity:

\[
p_t^{inn} = 1 - e^{-z L_{t-1}^E} \quad (36)
\]

Firms define the number of engineers they wish to employ as a ratio \(\nu^k\) of workers, constrained by the share \(\rho\) of cumulated profits they allocate to
**R&D:**

\[
L_t^E = \min \left\{ \nu^k L_t^k; \max \left\{ \rho^k R_t^E; 0 \right\} \right\} 
\]

(37)

The costs induced by the engineers pool is then included to the production cost of the next period.

The actual R&D follows a stochastic process of the following form:

1. Firms draw a number from a Uniform distribution on \([0; 1]\).
2. If this number is contained in the interval \([0; \rho_{inn}]\), the R&D is successful.
3. If R&D is successful, the characteristics of the newly developed vintage are randomly drawn as follows

\[
a_{\tau} = a_{\tau-1} (1 + \max \{\varepsilon_{\tau}^a; 0\})
\]

(38)

\[
\varepsilon_{\tau}^a \sim N(0; \sigma^a)
\]

(39)

**A.2.4 Minimum wage**

The minimum wage \(w_m^n\) follows an outward shifting wage curve of the form

\[
\Delta w_m^n = \begin{cases} 
\epsilon U \Delta U m_t & \text{IF } \overline{A}_{\Delta t} \leq \overline{A}_{\Delta t_0} \Omega^A \text{ or } \overline{P}_{\Delta t} \leq \overline{P}_{\Delta t_0} \Omega^P \\
- \epsilon U \Delta U m_t + \epsilon A \Delta \overline{A}_{\Delta t} + \epsilon P \Delta \overline{P}_{\Delta t} & \text{IF } \overline{A}_{\Delta t} > \overline{A}_{\Delta t_0} \Omega^A \& \overline{P}_{\Delta t} > \overline{P}_{\Delta t_0} \Omega^P
\end{cases}
\]

(40)

where \(\Delta U m_t = \frac{U m_t}{U m_{t-1}} - 1\) is clearly the variation in unemployment and \(\Delta \overline{A}_{\Delta t}\) and \(\Delta \overline{P}_{\Delta t}\) are respectively the variation in labour productivity and consumable prices. The \(\epsilon \in (0, 1)\) are the corresponding elasticities for the the effect of a change in the three variables on the minimum wage. \(^9\) While the *if conditions* on the left side simply define the wage curve shifts: when the labour productivity *and* the consumable prices in any period \(t\) have increased, respectively, by \(\Omega^A\) and \(\Omega^P\), with respect to the previous wage negotiation in period \(t_0\), a new discrete increase in the minimum wage in bargained.

Changes in productivity and consumable prices are accounted with respect to moving averages, assuming that the bargaining bodies consider their

\(^9\)Where \(\epsilon^U\) is empirically stable across time (Blanchflower and Oswald, 2006; Nijkamp and Poot, 2005).
trends and overlook schizophrenic and cyclical changes due to short run reactions to shocks, and perceive recent changes as more relevant. Change in prices is computed as:

\[ P_t = d\bar{p}_{t-1} + (1 - d) P_{t-1} \]  

(41)

where \( \bar{p}_{t-1} = z_{i,t-1}p_{i,t-1} \) is the weighted average of prices in the consumers market, \( z_{i,t-1} \) the market share of firm \( i \), and \( d \in (0,1) \) is a memory factor that defines the weight of past values on present decisions.

Similarly, observed changes in productivity are perceived as

\[ A_{a_t} = dA_{t-1} + (1 - d) A_{a,t-1} \]  

(42)

where

\[ A_{t-1} = \frac{Q_{t-1} + Y_{k,t-1}}{L_{t-1} + L_{t-1}^k + L_{t-1}^E} \]  

(43)

Finally, provided we do not have a population dynamic, the level of unemployment is computed using the available estimates of Beveridge curves (Wall and Zoega, 2002; Nickell, Nunziata, Ochel, and Quintini, 2002; Teo, Thangavelu, and Quah, 2004) defined as

\[ Um_t = C_L - \beta V_{t-1} \]  

(44)

\[ V_t = dV_{t-1} + (1 - d) V_{t-1} \]  

(45)

where \( C_L \) is a constant, \( \beta \) defines the shape of the curve, and \( V_{t-1} = V_{L,t-1} + V_{k,t-1} + V_{E,t-1} \) is the number of vacancies in the previous period computed as follows for the different types of employment:

\[ V_{L,t-1} = \sum_{l=1}^{\Lambda} V_{1,l,t-1}^{L} \]

\[ V_{L,t-1}^{L} = \left(1 + u^l\right) \frac{1}{A_{l-2}} \min\{Q_{t-1}^l; \bar{B}K_{t-2}\} \]

\[ V_{k,t-1} = \sum_{l=1}^{\Lambda} V_{1,l,t-1}^{K} \]

\[ V_{k,t-1}^{K} = (1 + u^m) \frac{K_{t-1}^k}{A^k} \]

\[ V_{E,t-1} = L_{t-1}^E. \]

\[ \text{Due to technical reasons we actually use an hyperbolic form of the Beveridge curve estimation } U_t = C^H + \beta/V_{t-1} \text{ for which B"orsch-Supan (1991) provide actual estimates on the German labour market.} \]
The sluggishness in the labour market and hiring process ($\epsilon_L$ and $\epsilon_K$ in the labour demand equations)\(^{11}\) determines the difference between open vacancies and actual number of workers.

Given the variety of estimated $\beta$s provided by the different authors across countries, datasets and econometric specifications, we use intermediate shapes of the Beveridge curve, which have also been found by Fagiolo, Dosi, and Gabriele (2004) in their Montecarlo simulations.

\(^{11}\text{Which can also be interpreted as the labour market friction that in matching models determine the level of unemployment as a function of the number of matches and vacancies. In our model the number of matches correspond to the workers actually hired (funtion of } \epsilon_L \text{ and } \epsilon_K \text{).}\)